Grade 5 Mathematics, Quarter 2, Unit 2.1

Add and Subtract Fractions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use equivalent fractions as a strategy to add fractions including mixed numbers.
- Use equivalent fractions as a strategy to subtract fractions including mixed numbers.
- Use fraction models to solve word problems involving addition and subtraction of fractions with like and unlike denominators.
- Use equations to solve real-world problems involving addition and subtraction of fractions with like and unlike denominators.
- Estimate the reasonableness of answers by using benchmark fractions.
- Estimate the reasonableness of answers by using number sense of fractions.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- State justification.
- Critique/defend.
- Ask questions to clarify an argument.

Model with mathematics.

- Draw conclusions, interpret results, and revise models if needed.
- Simplify the situation to a simpler rule allowing for revisions.
- Solve problems in everyday life through the use of algorithms, geometry, a function, etc.

Look for and express regularity in repeated reasoning.

- During the process, ask, "Does my work make sense?" and at the conclusion, "Does my answer make sense?"
- Make accurate calculations.

Essential questions

- When solving word problems involving addition and subtraction of fractions, why is it important to refer to the same whole?
- How do you use equivalent fractions to add any two fractions?
- How do you use equivalent fractions to subtract any two fractions?
- How would you use benchmark fractions to determine the reasonableness of answers when adding and subtracting fractions with unlike denominators?
- How do you determine common denominators?
- When adding or subtracting fractions with unlike denominators, why is it necessary to find common denominators?
- When adding or subtracting fractions with unlike denominators, how does number sense help in determining the reasonableness of answers?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions **5.NF**

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships

mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 3, students acquired comprehension of unit fractions, using visual fraction models to represent parts of the whole. They started to understand the relationship of part to whole. Students located a fraction on a number line between 0–1. They compared equivalent fractions on a number line, identified and produced simple equivalent fractions using models, and identified and wrote whole numbers as fractions. Students compared fractions using symbols > , < , = and justified their conclusions.

In grade 4, students discussed the difference in the number and size of the parts while telling why fractions are equivalent, using this method to identify and make equivalent fractions. Students made common denominators/numerators or compared to benchmark fractions with unlike denominators/numerators using visual models, symbols, and justifying solutions. Students in grade 4 also understood addition and subtraction of fractions as linking and breaking the same whole and decomposed fractions into equations using models, added and subtracted mixed numbers, and replacement equivalent fractions. They solved word problems with the same whole, like denominators, fractions models, and equations.

Current Learning

Students use their understanding of fraction equivalence and their skill in generating equivalent fractions as a strategy to add and subtract fractions with like and unlike denominators. They solve word problem involving the addition and subtraction of fractions with like and unlike denominators by using visual fraction models or equations to represent the problem. They also mentally estimate and assess the reasonableness of their answers. For example, they recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Future Learning

The heading will change from Number and Operations—Fractions to Number Systems. Students will use their understanding of fractions, and mesh it with their understanding of multiplication and division to apply it in situations where fractions are divided by fractions. Students will clarify and calculate fraction quotients and solve word problems with division of fractions using fraction models and equations.

Additional Findings

According to *Principles and Standards for School Mathematics*, "As students acquire conceptual grounding related to rational numbers, they should begin to solve problems using strategies they develop or adapt from their whole-number work. At these grades, the emphasis should not be on developing general procedures to solve all decimal and fraction problems. Rather, students should generate solutions that are based on number sense and properties of the operations and that use a variety of models or representations" (p. 155).

Grade 5 Mathematics, Quarter 2, Unit 2.2

Understanding Multiplication to Multiply Fractions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Interpret a fraction as the division of the numerator by denominator.
- Multiply a fraction by a whole number using visual models, stories, or equations.
- Multiply a fraction by a fraction using visual models, stories, or equations.
- Understand that the product of a fraction and a whole number can be separated into equivalent parts.
- Find the area of a rectangle with fractional side lengths.
- Solve problems using the multiplication of fractions using visual models, stories, or equations.

Mathematical practices to be integrated

Model with mathematics.

- Simplify the situation to a simpler rule allowing for revisions.
- Draw conclusions, interpret results, and revise models if needed.

Look for and make use of structure.

- Look for patterns to simplify.
- Use a variety of strategies and properties to verify answers.

Essential questions

- What does a fraction bar represent?
- How would you show ¼ can be written as 1 divided by 4 using manipulatives or pictures?
- How can you represent the multiplication of fractions?
- How can you find the area of a rectangle including fractional lengths?
- How an you use the multiplications of fractions to solve real-world problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 4, students began multiplying fractions by whole numbers. They used visual models to show the whole number times the fraction and recorded the result in an equation. They comprehend that multiples of the whole number and/ or multiples of the fraction compute to the same product when multiplied. (Ex. 2 x 2/3 = 4/3; 4 x 1/3 = 4/3). Students unraveled word problems that use multiplication of a fraction by a whole number using models and equations.

Current Learning

Students in grade 5 understand that a fraction is also a division problem, or the numerator divided by the denominator. Students use fraction models or equations to represent a given word problem. They divide whole numbers leading to quotients in the form of fractions or mixed numbers. Students focus their learning on modeling the multiplication of fractions of whole numbers and fractions. Students should continuously break apart fractions using number lines and fraction bars to discover generalized methods. They also find the area of a rectangle with fractional side lengths by tiling it with unit squares and multiplying the side lengths to show procedural equivalence.

Future Learning

In grade 6, the heading will change from Number and Operations—Fractions to Number Systems. Students in grade 6 will deduce and calculate quotients of fractions and decipher word problems that use division of fractions by fractions employing models and equations to exemplify the problem.

Additional Findings

For examples, see pp. 11–12 of the *Progressions for the Common Core State Standards in Mathematics*

According to *Principles and Standards for School Mathematics*, "Through teacher-orchestrated discussions of problems in context, students can develop useful methods to compute with fractions … in ways that make sense. Students' understanding of computation can be enhanced by developing their own methods and sharing them with one another, explaining why their methods work and are reasonable to use, and then comparing their methods with the algorithms traditionally taught in school. In this way, students can appreciate the power and efficiency of the traditional algorithms and also connect them to student–invented methods that may sometimes be less powerful or efficient but are often easier to understand" (p. 220).

Grade 5 Mathematics, Quarter 2, Unit 2.3

Interpreting Multiplication as Scaling or Resizing

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Estimate the size of a product compared to the size of one factor on the basis of the size of the other factor without multiplying.
- Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number.
- Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

Mathematical practices to be integrated

Look for and make use of structure.

- Look for patterns to simplify.
- Use a variety of strategies and properties to verify answers.

Look for and express regularity in repeated reasoning.

- During the process, ask, "Does my work make sense?" and at the conclusion, "Does my answer make sense?"
- Make accurate calculations.
- Look for repeated calculations and patterns.

Essential questions

- Why does multiplying a given number by a fraction less than 1 result in a product smaller than the given number?
- Why does multiplying a given number by a fraction greater than 1 result in a product greater than the given number?
- How can you use compatible numbers to estimate products of a fraction?
- How can resizing or scaling be explained using pictures?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4, students articulated equivalent fractions with denominators of 10 to add unlike denominators of 10 and 100. In addition, they named fractions with denominators of 10 and 100 in decimal notation and were able to position them on a number line. While defending their thinking, students made size comparisons using symbols and visual models when talking about the same whole.

Current Learning

In grade 5, students build on abstract reasoning and estimation as they mentally connect the size of a product on the foundational size of both factors acting on each other during the multiplication process. In addition, students explain this process and understand why one fraction generates greater or lesser results; they connect this to their knowledge of equivalent fractions.

Future Learning

In grade 6, students will use their thinking and understanding about fractions, scaling, multiplication, and division to decipher problems involving ratio and rate. Students will connect equivalent ratios and rates to the multiplication table. They will evaluate drawings that demonstrate relationships in scaling and size. All of this links ratios to fractions, while using multiplication and division to solve problems.

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics*:

In preparation for grade 6 work in ratios and proportional reasoning, students learn to see products such as 5 x 3 or 1/2 x 3 as expressions that can be interpreted in terms of a quantity, 3, and a scaling factor, 5 or 1/2. Thus, in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is 5 times as big as 3, without evaluating the product. Likewise, they see 1/2 x 3 as half the size of 3. The understanding of multiplication as scaling is an important opportunity for students to reason abstractly. Previous work with multiplication by whole numbers enables students to see multiplication by numbers bigger than 1 as producing a larger quantity, as when a recipe is doubled, for example. Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity, as when the budget of a large state university is multiplied by 1/2, for example. The special case of multiplying by 1, while leaving a quantity unchanged, can be related to fraction equivalence by expressing 1 as n/n. (p. 12)

Grade 5 Mathematics, Quarter 2, Unit 2.4

Understanding Division to Divide Fractions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Interpret division of a unit fraction by a non-zero whole number and compute quotients.
- Use visual fraction models to divide a unit fraction by a non-zero whole number (ex. ½ ÷ 4).
- Solve an equation dividing a unit fraction by a non-zero whole number (ex. 1/2 ÷ 4).
- Create a story for dividing a unit fraction by a non-zero whole number.
- Use a visual model to divide a whole number by a unit fraction (ex. 4 ÷ 1/5).
- Solve an equation to divide a whole number by a unit fraction.
- Create a story for dividing a whole number by a unit fraction.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify and execute appropriate strategies to solve the problem.
- Analyze given information to develop possible strategies for solving the problem.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Change perspectives, generate alternates and consider different options.

Model with mathematics.

- Analyze mathematical relationships to draw conclusions.
- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

Essential questions

- How does know how to multiply fractions help you to develop strategies to divide fractions?
- How would you divide a unit fraction by a whole number?
- Given the equation 1/2÷4=1/8, create a visual model to show understanding.
- How would you divide a unit fraction by a whole number?
- What is a situation where a unit fraction is divided by a whole number?
- How can you create a visual model to show that the equation 4÷1/3=12 is correct?
- What is a situation where a whole number is divided by a unit fraction?
- When would you create real-word situations for a division equation of a fraction by a whole number?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 3 students partition shapes into equal parts with equal areas and express this as a unit fraction of the whole. They also write whole numbers as fractions and understand that whole numbers are equivalent to fractions. They are also interpreting whole number quotients as sharing by partitioning them equally among groups.

In grade 4, students understand fractions as multiples and multiply fractions by whole numbers.

Current Learning

Grade 5 students will divide unit fractions by non-zero whole numbers and whole numbers by unit fractions. They have previous understanding from work in both grades three and four that taught whole numbers are equivalent to fractions and can be written as n/1. This is at the drill/practice level of instruction. Students also began multiplying fractions by whole numbers in grade four. Previously in grade 5, they multiplied fractions by fractions and multiplied and divided decimals and whole numbers. Since students are already able to multiply fractions by whole numbers and other fractions, they have the ability and mental thinking to relate multiplication and division. This will enable them to create strategies to divide fractions. However, dividing fractions by fractions is not required until grade 6. Division of fractions by non-zero whole numbers is at the reinforcement level because there have been other skills

like division and multiplication of whole numbers and multiplication of fractions that have prepared students to add this to their repertoire.

Future Learning

In grade 6, students divide fractions by fractions using visual fraction models and equations to represent the problem.

Grade 7 students will extend their understanding of division of fractions to divide rational numbers.

Additional Findings

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of the procedures for multiplying and dividing fractions and explain why they work. (*Curriculum Focal Points*, NCTM, p. 18)

Representing numbers with various physical materials should be a major part of math instruction in the elementary grades. (*Principles and Standards for School Mathematics*, p. 33)

Part of being able to compute fluently means making smart choices about which tools to use and when. (*Principles and Standards for School Mathematics*, p. 35)

Grade 5 Mathematics, Quarter 2, Unit 2.5

Solving Problems Involving Multiplication and Division of Fractions and Mixed Numbers

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve real world problems involving multiplication of fractions using visual models.
- Solve real world problems involving the multiplication of fractions using equations.
- Solve real world problems involving the multiplication of mixed numbers using visual models.
- Solve real world problems involving the multiplication of mixed numbers using equations.
- Divide whole numbers by unit fractions.
- Interpret division of a unit fraction by a whole number.
- Interpret division of a whole number by a unit fraction using visual models and equations.
- Solve real world problems involving division of fractions and whole numbers using visual models and equations.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify and execute appropriate strategies to solve the problem.
- Check their answers using a different method, and continually ask "Does this make sense?"

Construct viable arguments and critique the reasoning of others.

- Understand and use prior learning in constructing arguments
- Justify their conclusions, communicate them to others, and respond to the arguments of others.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

Essential questions

- How can you solve real world problems involving multiplication of fractions?
- How can you solve real world problems involving multiplication of mixed numbers?
- How can visual fraction models be used to solve real world problems?
- How can equations be used represent real world problems?
- How would you create a visual fraction model to show the division of a unit fraction by a nonzero whole number?
- How would you divide a nonzero whole number by a unit fraction?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 4, students disentangle word problems comprised of multiplication of a fraction by a whole number using visual models, as well as equations to signify the problem. Students in grade 4 apply their knowledge of division models and the relationship of multiplication to division.

Current Learning

Students in grade 5 extend their knowledge. They unscramble authentic challenges containing multiplication of fractions and mixed numbers using visual models and equations to symbolize the problem. They will also harness and broaden their understanding of division to encompass division of unit fractions by non zero whole numbers, whole numbers, and the inverse (unit numbers by unit fractions. The relationship between multiplication and division may help students formulate strategies to divide fractions. Division of a fraction by a fraction is not required. This is a critical area of instruction. Although the multiplication is at the reinforcement level, the rest is at the developmental level.

Future Learning

In grade 6, students use their knowledge of solving real world problems using multiplication and division to understand the ratio and rate and solve real world problems.

Additional Findings

Progressions for the Common Core State Standards in Mathematics states:

"Students use story problems to make sense of division problems;

How much chocolate will each person get if three people share 1/2 lb of chocolate equally? How many 1/3 cup servings are in two cups of raisins?

Students attend carefully to the underlying unit quantities when solving problems. For example, if one half of a fund raisers funds were raised by the sixth grade, and if one third of the sixth grades funds were raised by Ms. Wilkin's class, then 1/3 x 1/2 gives the fraction fund raisers funds that Ms. Wilkin's class raised but it does not tell us how much money Ms. Wilkin's class raised." (p. 12)